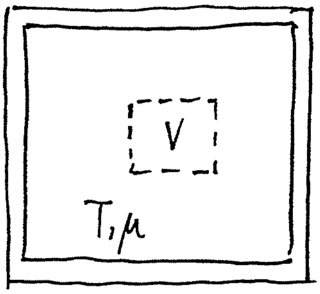
Formulation Based on Grand Partition Function



$$P(E_i(N),N) = \frac{e^{-\beta E_i(N) + \beta\mu N}}{Q(T,\mu,V)}$$

Gibbs distribution

$$Q(T,\mu,V) = \sum_{N=0}^{\infty} \sum_{\substack{N\text{-particle} \\ \text{states } i}} e^{-\beta E_i(N) + \beta\mu N}$$

Grand Partition Function

Grand potential

$$\Omega(T,\mu,V) = -kT \ln Q(T,\mu,V)$$

$$d\Omega = -S\,dT - N\,d\mu - p\,dV$$

$$\langle N\rangle = -\frac{\partial\Omega}{\partial\mu} = \frac{1}{\beta}\frac{\partial \ln Q}{\partial\mu}$$

$$S = -\frac{\partial\Omega}{\partial T} \quad ; \quad p = -\frac{\partial\Omega}{\partial V}$$

$$\langle E\rangle = \mu\langle N\rangle - \frac{\partial}{\partial\beta}\ln Q$$

$$\Omega = -pV$$

Thus, $pV = -\Omega = kT \ln Q$

- <u>Completely General!</u>

  i.e., Particles in System could be interacting

This is the Grand Canonical Ensemble approach.

In Ch. VII, we obtained the Fermi-Dirac and Bose-Einstein distributions as the most probable distributions. We also obtained the following equations for non-interacting fermions and bosons.

Fermions

$$f_{FD}(\varepsilon) = \frac{1}{e^{\alpha} e^{\beta\varepsilon} + 1} = \frac{1}{e^{\beta(\varepsilon-\mu)} + 1}$$

$$N = \sum_{\substack{\text{cells OR} \\ \text{levels } r}} g_r \frac{1}{e^{\beta(\varepsilon_r-\mu)} + 1} = \sum_{\text{all s.p. states } i} \frac{1}{e^{\beta(\varepsilon_i-\mu)} + 1}$$

$$E = \sum_{\substack{\text{cells OR} \\ \text{levels } r}} g_r \frac{\varepsilon_r}{e^{\beta(\varepsilon_r-\mu)} + 1} = \sum_{\text{all s.p. states } i} \frac{\varepsilon_i}{e^{\beta(\varepsilon_i-\mu)} + 1}$$

$$\sum_{\text{all s.p. states } i} (\cdots) \rightarrow \int g(\varepsilon)(\cdots)\, d\varepsilon \quad (\text{Ch. VIII})$$

Bosons

$$f_{BE}(\varepsilon) = \frac{1}{e^{\alpha} e^{\beta\varepsilon_r} - 1} = \frac{1}{e^{\beta(\varepsilon-\mu)} - 1}$$

$$N = \sum_{\substack{\text{cells OR} \\ \text{levels } r}} g_r \frac{1}{e^{\beta(\varepsilon_r-\mu)} - 1} = \sum_{\text{all s.p. states } i} \frac{1}{e^{\beta(\varepsilon_i-\mu)} - 1}$$

$$E = \sum_{\substack{\text{cells OR} \\ \text{levels } r}} g_r \frac{\varepsilon_r}{e^{\beta(\varepsilon_r-\mu)} - 1} = \sum_{\text{all s.p. states } i} \frac{\varepsilon_i}{e^{\beta(\varepsilon_i-\mu)} - 1}$$

$$\sum_{\text{all s.p. states } i} (\cdots) \rightarrow \int g(\varepsilon)(\cdots)\, d\varepsilon \quad (\text{Ch. VIII})$$

In Ch. XII, these equations are re-derived based on the grand partition function $\mathcal{Q}(T, V, \mu)$.